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Stable Matching: A heteronormative mathematical bijection

In this paper I analyze the language and discourse used in several texts concerning the stable matching problems, which are primarily studied in the areas of Computer Science, Mathematics, and Economics. In particular, I will focus on the "one-to-one, two-set, stable matching" problem, commonly known as the "stable marriage" problem. I will also discuss one of its most famous solution algorithms.

As a brief introduction, the "stable marriage" problem aims to find a bijection across two sets of the same cardinality, $A = \{m_1, m_2, \ldots m_n\}$ and $B = \{w_1, w_2, \ldots w_n\}$. The bijection must take into consideration the fact that each element in set *A* has a list of preferences over the elements of set *B*, and vice versa. Furthermore, each individual element only cares about satisfying its own interests. The ultimate goal is to find an algorithm that produces a bijection that constitutes a "stable matching". An *unstable* matching occurs whenever there exist two pairs, *mw* and *m'w'*, in the bijection such that *w* prefers *m'* over *m*, and *m'* also prefers *w* over *w'*, and therefore *w* and *m'* both have a reason to reject the current matching and form their own alliance[1].

Although in the paragraph above I have made an effort to describe the "one-to-one, two-set, stable matching" problem in terms of "meaningless" mathematical sets *A* and *B*, historically the "stable marriage" problem has usually been described in terms of matchmaking between a set of men and a set of women. This particular matching problem was first introduced by David Gale and Lloyd S. Shapley in their 1962

[1] Given that I did not formulate the original stable matching problems, I need to cite this definition somehow. The 2016 encyclopedia entry by Irving has a very similar definition. However, I formulated this particular version myself because I wanted it to be "neutral" but every other source I found was written in terms of men, women and marriage (or at best in terms of universities and applicants).

publication *College Admissions and the Stability of Marriage*. In their paper Gale and Shapley (G&S) primarily seek to find a stable matching algorithm for the one-to-many, two set problem of matching applicants to universities. However, in order to achieve this goal, they first designed a simpler special case in which there can only be one applicant admitted to each university.

Nevertheless, G&S claim that this one-to-one situation is "highly unnatural in the context of college admissions", and therefore introduce "another 'story' into which it fits quite readily" (G&S, p. 387). This, of course, is the story of a town with *n* women and *n* men, in which everyone "ranks those of the opposite sex in accordance with his or her preferences for a marriage partner" (G&S, p. 388). The goal is then to marry everyone off in such a way that there is no pair of a man and a woman who prefer each other more than they prefer their assigned "mates" (G&S, p. 388). By claiming that the "story" of one student for each university is "unnatural", the discourse of G&S seems to be implying that the "natural" alternative is to present a town in which everyone is heterosexual, and eager to be married into a monogamous relationship.

After introducing the "marriage problem" as described above, G&S move on to present the "deferred-acceptance" algorithm (commonly known today as the Gale-Shapley algorithm), which is guaranteed to find a stable matching given any possible input. The steps of the algorithm are quite simple to understand, as well as alarmingly covered by gender roles and social expectations. In the first round of the algorithm, each boy proposes to his first choice of girl, and each girl who was lucky enough to get one or more proposals chooses her favorite among her options, and puts

him on hold in case someone better proposes later. In the second round, boys who were rejected during the first round propose to their second choice of girl, and each girl chooses her favorite among her current boy and her new proposals. This process continues until every girl has put some boy on hold, in which moment all of the girls are "required to accept" whichever husband they got (G&S, p. 389).

The G-S algorithm perpetrates the social dictations of women as passive and men as active. Moreover, all the boys get to shoot for the stars and ask out the girls in order of preference, while the girls are required to wait for proposals and settle with whatever boy they get. Even more impressive is the fact that we can mathematically prove that this algorithm is man-optimal and woman-pessimal (G&S, p.390). Of course, every source[2] is quick to remind us that, if the roles were inverted and women were the ones proposing to men, the algorithm would become woman-optimal. However, very few sources[3] actually choose to present the women-proposing variation as default. Furthermore, the act of realizing that this algorithm favors so much one side over the other, and still choosing to use marriage as an illustration, suggests that G&S did not care at all about what social implications their publication may have had.

One may argue that G&S were men born in the 1920's and therefore lived in a context that was far more heteronormative and sexist than it is today. Thus it is understandable that they came up with this particular illustration and published it without a second thought. Nevertheless, it seems like the same illustration is still being used nowadays. The only source I could find that did not use the man-woman concept at all,

[2] Literally every source in my bibliography that mentions marriage does this.
[3] In my bibliography only Numberphile and the Nobel Prize webpage.

and instead opted to go back to the “unnatural” single-applicant-per-university story was the 1981 paper by Lester E. Dubins and David A. Freedman (D&F) titled *Machiavelli and the Gale-Shapley Algorithm*. In summary, this paper shows that, when using the G-S algorithm, the proposing party, in this case the applicants, cannot get any benefit from faking their preferences (D&F, p. 487). Alternatively, the party being proposed to, in this case the universities, could sometimes do better by lying about their preference (D&F, p. 493).

Although the words “marriage”, “men”, and “women” do not appear even once in *Machiavelli and the [G-S] Algorithm*, whenever other sources bring up D&F’s results, they still do it in context of the “marriage problem”. There is a set of presentation slides created by Kevin Wayne for an introductory lecture in Analysis of Algorithms that contains a single slide titled “Deceit: Machiavelli meets Gale-Shapley”. This slide briefly summarizes D&F’s results in terms of men and women (Wayne, s. 21), and it never mentions that the original paper was actually written in terms of applicants and universities. Similarly, a set of slides by Emily Riehl, created for an introductory course in Game Theory, mentions D&F’s results on a single slide titled “Men shouldn’t lie” (Riehl, s.17), and also fails to point out that the original paper was not in terms of men, women or marriage.

Even though the two sets of slides are structured in a very similar way, they actually have some subtle differences. Some of Riehl’s slide titles tend to hint more towards the social implications of the topic, while Wayne’s slide titles are more literal and directly related to the technical aspects of the algorithm. For instance, while Riehl’s

slide on the corollary for woman-pessimality is titled “Sexism in the male-proposing algorithm” (Riehl, s. 15), Wayne simply titles it “Woman-pessimality” (Wayne, s. 20). Similarly, while Riehl’s slide dedicated to the single-set variation of the problem is titled “Heteronormativity” (Riehl, s. 8), Wayne chooses the usual name for that variation, which is the “Stable roommate problem” (Wayne, s. 8). Nevertheless, there is one slide at the end of Wayne’s presentation dedicated to “Lessons learned” with a sub-title that reads “Potentially deep social ramifications (legal disclaimer)”. Among other things, this slide encourages men to be more honest, and women to be brave and ask out the men (Wayne, s.25).

Even though it is evident that this “legal disclaimer” is Wayne’s way to address the social implications of the “stable marriage” topic, it is not possible to grasp how serious he is about these topics without listening to his lecture[4]. Fortunately, in the case of Emily Riehl I was able to understand more of her views due to a pair of videos uploaded to youTube by the mathematics account *Numberphile*. In these videos, Riehl chooses to present the G-S algorithm in the woman-proposing mode, not without stating that the original version had the men proposing because “this was the 60’s after all” (Numberphile2, 6:11-6:17). She also mentions that the marriages need to be heterosexual in order for the mathematics to work out (Numberphile, 0:43-0:17)[5]. Furthermore, she points out that the G&S “noticed right away” that the algorithm is man-optimal, but it took researchers around ten more years to realize it is also

[4] I did attend a lecture at UCSC once in which the professor was using the same set of slides. I remember him claiming that it was a “feminist lecture” (whatever that means). He also clarified that this was just an example and that he did not expect anyone to actually apply the algorithm in real life.
[5] The “stable roommates” (or “queer stable marriage” as we could start calling it) problem actually has no solution, as can be seen in either set of presentation slides (coincidentally slide 8 in both).

woman-pessimal “even though the argument [to prove woman-pessimality] is exactly the same [as proving man-optimality]” (Numberphile2, 6:17-6:37). This commentary of the social implications of the “marriage problem” is unlike any other source I could find.

Besides the *Numberphile* videos, the only other source I encountered that presented the G-S algorithm with the woman-proposing mode as default was the Nobel Prize’s “information to the public” document. This document describes the work of the 2012 Economics Prize Laureates, Lloyd Shapley and Alvin Roth (Nobel Prize, p. 1). The prize was awarded to both men due to the work Roth has done in adapting Shapley’s algorithm to real-world economic problems. An article in The New York Times by Catherine Rampell explains that the Laureates’ work primarily consists on “figuring out how to properly assign people and things to stable matchings, when prices are not available to help sellers and buyers pair up” (Rampell, pph. 2). This article is mainly focuses on the economic, real-life applications of the G-S algorithm, and mentions the “stable marriage” illustration in only one paragraph (Rampell, pph. 6).

In fact, Rampell mostly focuses on explaining how Roth’s work to adapt the “deferred-acceptance algorithm” can be applied to every-day life situations such as matching residents to hospitals (Rampell, pph. 14), applicants to schools (Rampell, pph. 7) or even kidney donors to patients (Rampell, pph. 16). Rampell quotes Dr. Parag Pathak, one of Roth’s colleagues, who explains that the G-S algorithm “levels the playing field” so that each candidate can get their best possible match by simply being truthful about their preference[6], and therefore no one is inclined to lie or device

[6] This is due to the intrinsic no-cheating feature of the algorithm demonstrated by Dubins and Freedman.

strategies in an attempt to play the system (Rampell, pph. 13). With this we can see that, when it comes to real-life adaptations, people don't seem to care about the initial premise of the algorithm anymore.

When looking at the economics approaches to the stable matching problem and the Gale-Shapley algorithm, we can notice that the original illustration of the "marriage problem" is hardly ever mentioned. Fifty-four years after Gale and Shapley published their original paper on *College Admissions and the Stability of Marriage*, their algorithm is still being used by diverse researchers, including Zixuan Peng et al. in their paper titled *Stable Vessel-Cargo Marching in Dry Bulk Shipping Market With Price Game Mechanism*. Peng et al.'s paper has absolutely nothing to do with marrying off a set of *n* men and *n* women in such a way that no one is motivated to elope. However the core functionality that both papers seek is still the same, that is, both papers aim to find a stable matching across two sets in an optimal and efficient way.

The field of Economics might have drifted away from the social implications of the "stable marriage" problem by borrowing only its most practical applications. However, this definitely doesn't mean that such implications have disappeared. Stable matching in the format of "stable marriage" is still a very common introductory topic to many Computer Science and Mathematics courses. Although most people are able to distinguish fantasy from reality, the way in which lecturers approach this topic can still reflect their own social perspectives. This could potentially become sensitive considering that most of these lectures are given in classrooms that are not very diverse.

Annotated Bibliography:

- Dubins, Lester E., and David A. Freedman. “Machiavelli and the Gale Shapley Algorithm.” *The American Mathematical Monthly* 88.7 (1981): 485-94. Web. - After seeing the “Machiavelli meets Gale-Shapley” slide in one of the presentations, I decide to look for the original paper to see how they talked about the subject. I was surprised to find one of the few sources that doesn’t mention “men”, “women”, or “marriage” at all. This is why I decided to include it in my paper. (SATISFIES: Scholarly article (1)).
- Gale, David, and Lloyd Stowell Shapley. "College Admissions and the Stability of Marriage." *The American Mathematical Monthly* 120.5 (2013): 386-91. Web. - This is where the Gale-Shapley algorithm, as well as the “Stable Marriage” illustration, first originated. I found the title of the article in the bibliography of another paper (that I ended up not using), and I looked for it at the university library’s database. Although this is a reprint from 2013 (probably due to Shapley and Roth winning the Nobel prize in 2012), the original article was actually published in 1962. This article is in the area of Mathematics and Computer Science and I thought it would be important to include because it is where the original idea came from. (SATISFIES: Scholarly article (2)).
- Irving, Robert W. "Stable Marriage." *Encyclopedia of Algorithms*. Ed. Ming-Yang Kao. New York: Springer, 2016. 2060-064. Print. - I came across this by typing “Stable Matching Encyclopedia” in a Google search. Then I found a pdf of letter ‘S’ at the university library’s database. “Stable *Matching*” doesn’t show up in

non-specialized encyclopedias, and in a sense it doesn't show up in algorithm encyclopedias either (all the entries I could find referred to "Stable *Marriage*").

NOTE: I didn't really end up using this source too much.

- Irving, Robert W., Paul Leather, and Dan Gusfield. "An Efficient Algorithm for the 'Optimal' Stable Marriage." *Journal of the ACM* (1987): 532-43. Web. - When doing an initial Google search of "Stable Matching" I came across a few pages of a book by Irving and Gusfield, consequently I looked up their names and came across the abstract of this article, which I thought was interesting. Then I looked for the article in the university library's database. This article is in the area of Computer Science, and I included it because, in words of the authors, "it uses a more equitable or egalitarian criterion of optimality".

  NOTE: I didn't end up using this source at all, but decided to keep it for future reference.

- "The Prize in Economic Sciences 2012 - Popular Information". *Nobelprize.org.* Nobel Media AB 2014. Web. 24 Apr 2017.- Click on "Public Information" pdf. <http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2012/popular.html> - I found this during my initial Google search of "Stable Matching". I thought it was interesting to include it because it is one of the few sources I found that introduces the Gale-Shapley algorithm with the women proposing to the men first ("women optimal"), and not the other way around.

- "Stable Marriage Problem - Numberphile." YouTube video, 8:38. Posted by "Numberphile," September 4, 2014. <http://youtu.be/Qcv1IqHWAzg> AND

"Stable Marriage Problem (the math bit)." YouTube video, 11:37. Posted by "Numberphile2," September 4, 2014. <https://youtu.be/LtTV6rIxhdo> - I found these videos during my initial Google search of "Stable Matching", I noticed the person talking is the author for the Solution to the Stable Marriage Problem lecture slides (Emily Riehl, Harvard University) and I thought some of the comments made during the videos were interesting. (SATISFIES: Webpage).

- Peng, Zixuan, Wenxuan Shan, Feng Guan, and Bin Yu. "Stable Vessel-Cargo Marching in Dry Bulk Shipping Market With Price Game Mechanism." *Elsevier: Transportation Research part E* 95 (2016): 76-94. Web. - I found this article in the university library's database by looking for "Stable Matching" and "Economics", since I wanted to find a paper from another discipline and the Nobel Prize awarded to Shapley and Roth was on Economics. I also wanted to include a more recent article to see how the initial idea of "stable matching" is being applied today, as well as how the concept of "stable marriage" is treated by recent researchers. (SATISFIES: Scholarly article (3)- different discipline).
- Rampell, Catherine. "2 From U.S. Win Nobel in Economics." *The New York Times*. N.p., 15 Oct. 2012. Web. 24 Apr. 2017. - I came across this newspaper article by searching for news about the Economics Nobel Prize of 2012 in Google (after finding out about it due to the nobelprize.org source above). I chose to include this article because it pays more attention to the economic aspects and applications of the algorithm, and only has one simple paragraph that mentions marriages. (SATISFIES: News Article).

- Riehl, Emily. (2013). A Solution to the Stable Marriage Problem. [Presentation Slides]. Retrieved from: <http://www.math.jhu.edu/~eriehl/pechakucha.pdf>. - I found this presentation when I was looking for the one created by Kevin Wayne (below). I thought it provided an interesting contrast with the other set of slides in the way it presents the problem, solution, and theorems, as well as how it addresses heteronormativity and sexism. (SATISFIES: Encyclopedia)
- Wayne, Kevin. (2005, last updated 2014). Algorithm Design: 1. Representative Problems. [Presentation Slides]. Retrieved from: <http://www.cs.princeton.edu/~wayne/kleinberg-tardos/pdf/01StableMatching.pdf> - These are the official lecture slides for a textbook titled “Algorithm Design” by Jon Kleinberg and Eva Tardos. Said textbook is used in the CMPS 102 Analysis of Algorithms class at UCSC, and that is how I found out about them. I chose to include these because of my personal experience in a class using these slides, as well as due to the interesting conclusions drawn at the end. (SATISFIES: Encyclopedia).